

32.5 A sous vide immersion cooker operates by way of a flat 6inch square vertical plate on one side of a water bath, maintaining the water temperature for many hours. Moments after being turned on, the plate is $140^{\circ}F$ and the water is $60^{\circ}F$. What is the rate of heat transfer? Refer to the table below for properties of water.

T ($^{\circ}F$)	ρ (lbm/ft ³)	c_p (Btu/lbm- $^{\circ}F$)	μ (lbm/ft-sec)	ν (ft ² /sec)	k (Btu/hr-ft- $^{\circ}F$)	Pr	β (1/ $^{\circ}F$)
32	62.4	1.01	1.20×10^{-3}	1.93×10^{-5}	0.319	13.7	-0.37×10^{-4}
40	62.4	1.00	1.04×10^{-3}	1.67×10^{-5}	0.325	11.6	0.20×10^{-4}
50	62.4	1.00	0.88×10^{-3}	1.40×10^{-5}	0.332	9.55	0.49×10^{-4}
60	62.3	0.999	0.76×10^{-3}	1.22×10^{-5}	0.340	8.03	0.85×10^{-4}
70	62.3	0.998	0.658×10^{-3}	1.06×10^{-5}	0.347	6.82	1.2×10^{-4}
80	62.2	0.998	0.578×10^{-3}	0.93×10^{-5}	0.353	5.89	1.5×10^{-4}
90	62.1	0.997	0.514×10^{-3}	0.825×10^{-5}	0.359	5.13	1.8×10^{-4}
100	62.0	0.998	0.458×10^{-3}	0.740×10^{-5}	0.364	4.52	2.0×10^{-4}
150	61.2	1.00	0.292×10^{-3}	0.477×10^{-5}	0.384	2.74	3.1×10^{-4}
200	60.1	1.00	0.205×10^{-3}	0.341×10^{-5}	0.394	1.88	4.0×10^{-4}

- A. $2540 \frac{Btu}{hr}$
- B. $3180 \frac{Btu}{hr}$
- C. $4620 \frac{Btu}{hr}$
- D. $5000 \frac{Btu}{hr}$

For convection, apply **Newton's Law of Cooling**:

$$\dot{Q} = hA\Delta T$$

For **Natural (Free) Convection** involving a **Vertical Flat Plate in Large Body of Stationary Fluid**, the convection heat transfer coefficient is:

$$\bar{h} = C \left(\frac{k}{L} \right) Ra_L^n$$

where C and n are constants, k is the thermal conductivity, L is the vertical height of the plate, and Ra_L is the **Rayleigh Number**, which can be calculated using:

$$Ra_L = \frac{g\beta(T_s - T_{\infty})L^3}{\nu^2} Pr$$

where g is acceleration due to gravity, β is the coefficient of thermal expansion for water, T_s is the average surface temperature of the plate, T_{∞} is the bulk (ambient) temperature of the water, L is the vertical height of the plate, ν is the kinematic viscosity, and Pr is the Prantl number. Use the table **Properties of Water** and the table provided to obtain Pr , ν , β , and k , then calculate Ra_L .

Calculate the film temperature for looking up various parameters required to determine the Rayleigh Number:

$$T_f = \frac{T_s + T_{\infty}}{2} = \frac{140^{\circ}F + 60^{\circ}F}{2} = 100^{\circ}F$$

$$Ra_L = \frac{\left(32.2 \frac{ft}{s^2}\right) \left(\frac{2 \times 10^{-4}}{^\circ F}\right) (140^\circ F - 60^\circ F) (.5ft)^3 (4.52)}{\left(.74 \times 10^{-5} \frac{ft^2}{s}\right)^2} = 5.32 \times 10^9$$

Based on the range of Ra_L select the appropriate constants C and n :

$$10^9 < Ra_L < 10^{13}$$

$$C = 0.10$$

$$n = 1/3$$

Calculate \bar{h} :

$$\bar{h} = C \left(\frac{k}{L}\right) Ra_L^n = (.1) \left(\frac{\left(.364 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}\right)}{(.5ft)}\right) (5.32 \times 10^9)^{\frac{1}{3}} = 127 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$$

Determine the rate of heat transfer, \dot{Q} :

$$\dot{Q} = hA\Delta T = \left(127 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}\right) (.5ft)^2 (140^\circ F - 60^\circ F) = 2541 \frac{Btu}{hr}$$

Answer A